



22077401

**MATHEMATICAL STUDIES  
STANDARD LEVEL  
PAPER 1**

Monday 7 May 2007 (afternoon)

1 hour 30 minutes

Candidate session number

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**INSTRUCTIONS TO CANDIDATES**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Five pipes labelled, “6 metres in length”, were delivered to a building site. The contractor measured each pipe to check its length (in metres) and recorded the following;

5.96, 5.95, 6.02, 5.95, 5.99.

- (a) (i) Find the mean of the contractor’s measurements.
- (ii) Calculate the percentage error between the mean and the stated, **approximate** length of 6 metres.
- (b) Calculate  $\sqrt{3.87^5 - 8.73^{-0.5}}$ , giving your answer
  - (i) correct to the nearest integer,
  - (ii) in the form  $a \times 10^k$ , where  $1 \leq a < 10, k \in \mathbb{Z}$ .

*Working:*

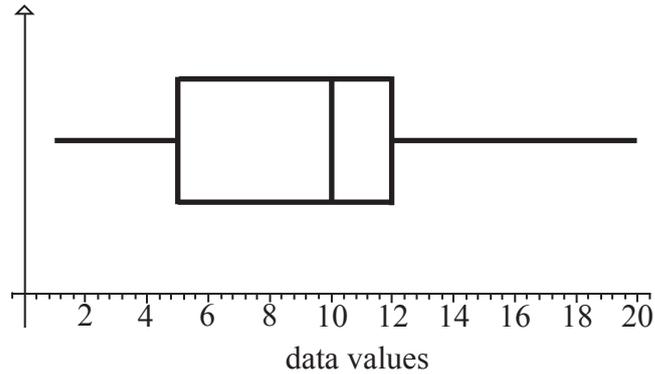
*Answers:*

- (a) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_
- (b) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_



2. (a) State which of the following sets of data are discrete.
- (i) Speeds of cars travelling along a road.
  - (ii) Numbers of members in families.
  - (iii) Maximum daily temperatures.
  - (iv) Heights of people in a class measured to the nearest cm.
  - (v) Daily intake of protein by members of a sporting team.

The boxplot below shows the statistics for a set of data.



- (b) For this data set write down the value of
- (i) the median
  - (ii) the upper quartile
  - (iii) the minimum value present
- (c) Write down three different integers whose mean is 10.

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_
- (iii) \_\_\_\_\_
- (c) \_\_\_\_\_



3. (a) 1 Brazilian Real (BRL) = 2.607 South African Rand (ZAR). Giving answers **correct to two decimal places**,
- (i) convert 300 BRL to ZAR,
  - (ii) find how many Real it costs to purchase 300 Rand.

Marilia deposits a gift of 150 Real from her aunt into a savings account. The savings account pays an annual simple interest rate of  $r\%$ . The interest is added to the account at the end of each month. After 9 months, the amount in the savings account was 158.10 Real.

- (b) Find the value of  $r$ .

*Working:*

*Answers:*

- (a) (i) \_\_\_\_\_  
(ii) \_\_\_\_\_  
(b) \_\_\_\_\_



4. The truth table below shows the truth-values for the proposition

$$p \underline{\vee} q \Rightarrow \neg p \underline{\vee} \neg q$$

$p$	$q$	$\neg p$	$\neg q$	$p \underline{\vee} q$	$\neg p \underline{\vee} \neg q$	$p \underline{\vee} q \Rightarrow \neg p \underline{\vee} \neg q$
T	T	F	F		F	
T	F	F		T	T	T
F	T	T	F	T	T	T
F	F	T	T	F		T

- (a) Explain the distinction between the compound propositions,  $p \underline{\vee} q$  and  $p \vee q$ .
- (b) Fill in the four missing truth-values on the table.
- (c) State whether the proposition  $p \underline{\vee} q \Rightarrow \neg p \underline{\vee} \neg q$  is a tautology, a contradiction or neither.

*Working:*

*Answers:*

- (a) \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- (c) \_\_\_\_\_



5. The table below shows the **monthly** repayments per \$10 000 borrowed for various nominal annual interest rates.

Loan term (years)	Table of monthly repayments in \$, per \$10 000		
	Annual interest rate		
	7%	8%	9%
5	198.0112	202.7634	207.5836
10	116.1085	121.3276	126.6758
15	89.8828	95.5652	101.4267
20	77.5299	83.6440	89.9726
25	70.6779	77.1816	83.9196

Beryl borrows \$150 000 to buy an apartment at an interest rate of 8%, to be repaid over 20 years.

- (a) Calculate Beryl’s exact monthly repayment.
- (b) Find the exact amount of **interest** paid for the loan over the 20 years.

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_



6. (a) Write down the following numbers in increasing order.

3.5,  $1.6 \times 10^{-19}$ , 60730,  $6.073 \times 10^5$ ,  $0.006073 \times 10^6$ ,  $\pi$ ,  $9.8 \times 10^{-18}$ .

(b) Write down the median of the numbers in part (a).

(c) State which of the numbers in part (a) is irrational.

*Working:*

*Answers:*

(a)

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(b)

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(c)

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7.  $B$  and  $C$  are subsets of a universal set  $U$  such that

$$U = \{x : x \in \mathbb{Z}, 0 \leq x < 10\}, B = \{\text{prime numbers} < 10\}, C = \{x : x \in \mathbb{Z}, 1 < x \leq 6\}.$$

(a) List the members of sets

(i)  $B$

(ii)  $C \cap B$

(iii)  $B \cup C'$

Consider the propositions:

$p$  :  $x$  is a prime number less than 10.

$q$  :  $x$  is a positive integer between 1 and 7.

(b) Write down, in words, the contrapositive of the statement, "If  $x$  is a prime number less than 10, then  $x$  is a positive integer between 1 and 7."

*Working:*

*Answers:*

(a) (i) \_\_\_\_\_

(ii) \_\_\_\_\_

(iii) \_\_\_\_\_

(b) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



8. The local park is used for walking dogs. The sizes of the dogs are observed at different times of the day. The table below shows the numbers of dogs present, classified by size, at three different times last Sunday.

	Small	Medium	Large
Morning	9	18	21
Afternoon	11	6	13
Evening	7	8	9

- (a) Write a suitable null hypothesis for a  $\chi^2$  test on this data.
- (b) Write down the value of  $\chi^2$  for this data.
- (c) The number of degrees of freedom is 4. Show how this value is calculated.

The critical value, at the 5% level of significance, is 9.488.

- (d) What conclusion can be drawn from this test? Give a reason for your answer.

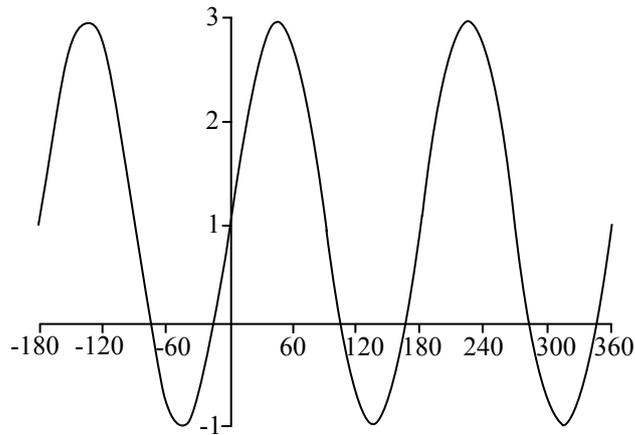
*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_
- (d) \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_



9. The graph of  $y = a \sin 2x + c$  is shown below,  $-180 \leq x \leq 360$ ,  $x$  is measured in degrees.



- (a) State:
  - (i) the period of the function,
  - (ii) the amplitude of the function.
- (b) Determine the values of  $a$  and  $c$ .
- (c) Calculate the value of the first **negative**  $x$ -intercept.

*Working:*

*Answers:*

- (a) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_



10.  $P(4, 1)$  and  $Q(0, -5)$  are points on the coordinate plane.

- (a) Determine the
  - (i) coordinates of  $M$ , the midpoint of  $P$  and  $Q$ .
  - (ii) gradient of the line drawn through  $P$  and  $Q$ .
  - (iii) gradient of the line drawn through  $M$ , perpendicular to  $PQ$ .

The perpendicular line drawn through  $M$  meets the  $y$ -axis at  $R(0, k)$ .

- (b) Find  $k$ .

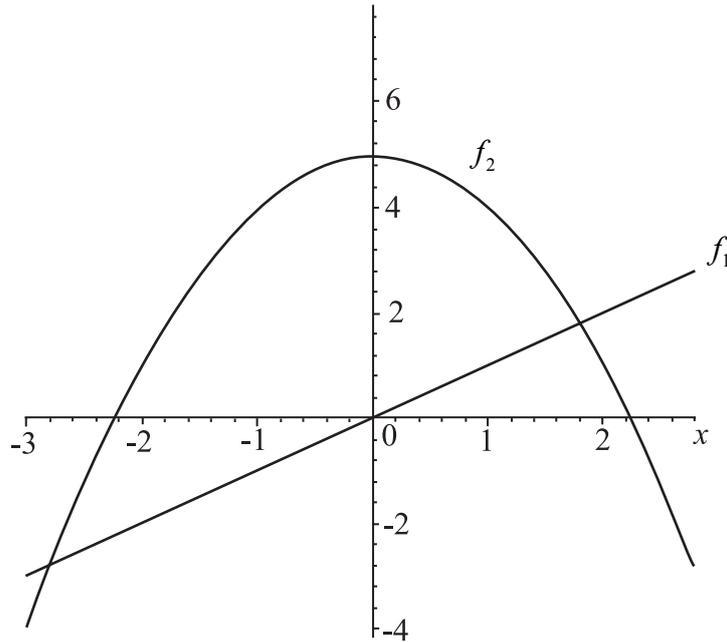
*Working:*

*Answers:*

- (a) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_
- (iii) \_\_\_\_\_
- (b) \_\_\_\_\_



11. The figure below shows the graphs of functions  $f_1(x) = x$  and  $f_2(x) = 5 - x^2$ .



- (a) (i) Differentiate  $f_1(x)$  with respect to  $x$ .
- (ii) Differentiate  $f_2(x)$  with respect to  $x$ .
- (b) Calculate the value of  $x$  for which the gradient of the two graphs is the same.
- (c) Draw the tangent to the **curved** graph for this value of  $x$  on the figure, showing clearly the property in part (b).

*Working:*

*Answers:*

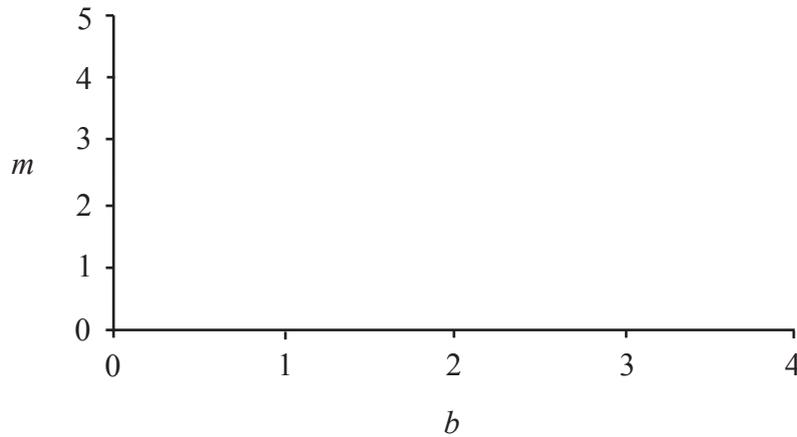
- (a) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_
- (b) \_\_\_\_\_



12. A store sells bread and milk. On Tuesday, 8 loaves of bread and 5 litres of milk were sold for \$21.40. On Thursday, 6 loaves of bread and 9 litres of milk were sold for \$23.40.

If  $b$  = the price of a loaf of bread and  $m$  = the price of one litre of milk, Tuesday's sales can be written as  $8b + 5m = 21.40$ .

- (a) Using simplest terms, write an equation in  $b$  and  $m$  for Thursday's sales.
- (b) Find  $b$  and  $m$ .
- (c) Draw a sketch, in the space provided, to show how the prices can be found graphically.



*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_

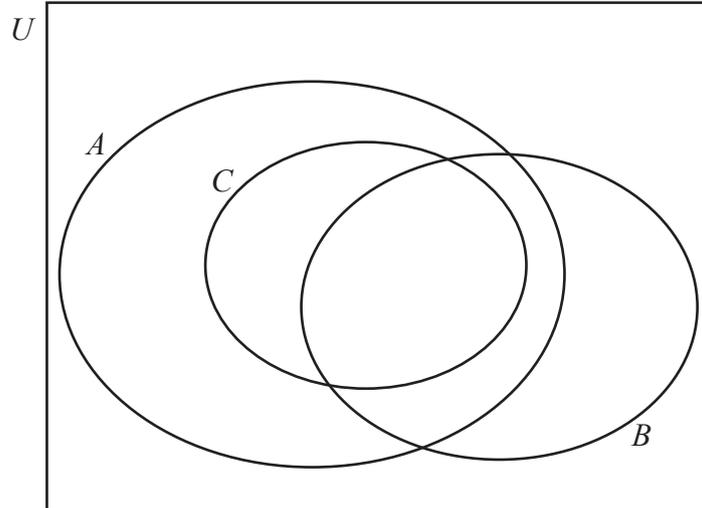


13. The figure below shows the set  $U$  of all functions of  $x$ .

$A$  is the set of all sine and cosine functions of  $x$  (measured in degrees).

$B$  is the set of all functions of  $x$  with period  $120^\circ$ .

$C$  is the subset of  $A$ , containing functions whose amplitude is 3.



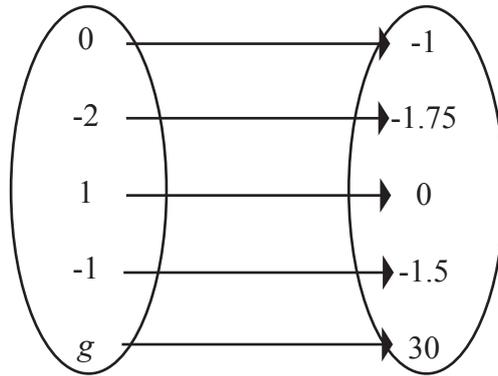
Write the following functions in the correct place in the Venn diagram.  
(You may use the numbers (i) to (vi) if you wish, to achieve a better fit.)

- (i)  $\sin(x)$ ,
- (ii)  $x^2$ ,
- (iii)  $3\cos(x)$ ,
- (iv)  $2\sin(3x)$ ,
- (v)  $3\cos(3x)$ ,
- (vi)  $-3\sin(2x)$ .

*Working:*



14. The mapping below is of the form  $f : x \mapsto a \times 2^x + b$  and maps the elements of  $x$  to elements of  $y$ .



- (a) (i) List the elements in the domain of  $f$ .
- (ii) List the elements in the range of  $f$ .
- (b) Find  $a$  and  $b$ .
- (c) Find the value of  $g$ .

*Working:*

*Answers:*

- (a) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_



15. The length of one side of a rectangle is 2 cm longer than its width.
- (a) If the smaller side is  $x$  cm, find the perimeter of the rectangle in terms of  $x$ .

The perimeter of a square is equal to the perimeter of the rectangle in part (a).

- (b) Determine the length of each side of the square in terms of  $x$ .

The sum of the areas of the rectangle and the square is  $2x^2 + 4x + 1$  ( $\text{cm}^2$ ).

- (c) (i) Given that this sum is  $49 \text{ cm}^2$ , find  $x$ .
- (ii) Find the area of the square.

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_

